$A(0,2,4)$
$B(1,0,2)$
$C(-4,2,1)$
} върхове на Δ

Търси се:
а) $P_\Delta = ?$; б) $\sphericalangle BAC = ?$
в) $H = ?$

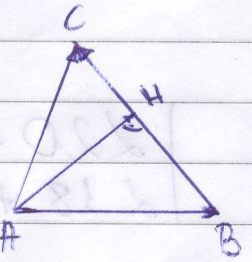
а) $\vec{AB} = (1,-2,-2)$
$\vec{AC} = (-4,0,-3)$
$\vec{BC} = (-5,2,-1)$

$|\vec{AB}| = \sqrt{1^2+2^2+2^2} = \sqrt{9} = 3$
$|\vec{AC}| = \sqrt{4^2+3^2} = \sqrt{25} = 5$
$|\vec{BC}| = \sqrt{5^2+2^2+1} = \sqrt{30}$

$$P_\Delta = 3+5+\sqrt{30} = 8+\sqrt{30}$$

б) $\cos \sphericalangle BAC = \dfrac{\vec{AB}\,\vec{AC}}{|\vec{AB}||\vec{AC}|} = \dfrac{-4+0+6}{3\cdot 5} = \dfrac{2}{15}$

в) $\vec{AB} + \vec{BH} = \vec{AH}$
$\vec{AB} + \lambda \vec{BC} = \vec{AH} \quad / \cdot \vec{BC}$
$\vec{AB}\vec{BC} + \lambda \vec{BC}^2 = 0$



$-7 + 30\lambda = 0 \Rightarrow \lambda = 7/30$
$\vec{BH} = 7/30\ \vec{BC}$

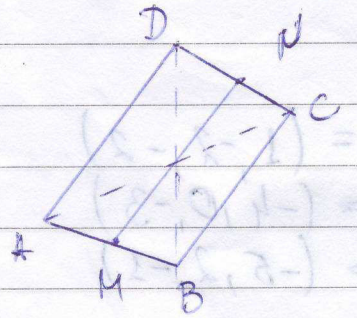
$H(x,y,z)$

$$\left.\begin{array}{l} x-1 = 7/30 \ /(-5) \\ y-0 = 7/30 \ /2 \\ z-2 = 7/30 \ /(-1) \end{array}\right\} \Rightarrow \left|\begin{array}{l} x = -1/6 \\ y = \frac{7}{15} \\ z = 53/30 \end{array}\right.$$

$A(0,0,2)$
$B(4,0,-4)$
$C(2,0,0)$
$D(5,3,-3)$

$M \in AB \quad N \in CD$

MN - ос отсека



$\vec{AM} + \vec{MN} + \vec{ND} = \vec{AD}$

$\alpha\vec{AB} + \beta\vec{CD} = \vec{AD}$

$$\begin{cases} \alpha\vec{AB}^2 + \beta\vec{AB}\vec{CD} = \vec{AD}\vec{AB} \\ \alpha\vec{AB}\vec{AC} + \beta\vec{CD}^2 = \vec{AD}\vec{CD} \end{cases}$$

$\vec{AB}(4\ 0\ -2)$
$\vec{CD}(3\ 3\ -3)$
$\vec{AD}(5\ 3\ -1)$

$\Rightarrow$

$\vec{AB}^2 = 20$
$\vec{AB}\vec{CD} = 18$
$\vec{AD}\vec{AB} = 22$

$\vec{CD}^2 = 27$
$\vec{AD}\vec{CD} = 27$

$$\begin{cases} \alpha 20 + \beta 18 = 22 \\ \alpha 18 + \beta 27 = 27 \end{cases} \Rightarrow \begin{matrix} \alpha = 1/2 \\ \beta = 2/3 \end{matrix}$$

$M(2\ 0\ -3) \quad \vec{ND} = 2/3\,\vec{CD} \quad N(x\ y\ z)$

$$\begin{cases} 5 - x = 2/3\,3 \\ 3 - y = 2/3\,3 \\ -3 - z = 2/3\,(-3) \end{cases}$$

$x = 3$
$y = 1$
$z = -1$

$A(1\ -5\ 4)$
$B(0\ -3\ 1)$
$C(-2\ -4\ 3)$
$D(4\ 4\ -2)$

Търси се
а) $V_a = ?$
б) $H_a = ?$

$\left.\begin{matrix}\overrightarrow{BA}\ (1\ -2\ 3)\\ \overrightarrow{BC}\ (2\ -1\ 2)\\ \overrightarrow{BD}\ (4\ -7\ -3)\end{matrix}\right\} \Rightarrow \overrightarrow{BA}\,\overrightarrow{BC}\,\overrightarrow{BD} = \begin{vmatrix}1 & -2 & 3\\ 2 & -1 & 2\\ 4 & -7 & -3\end{vmatrix} =$

$3 + (-16) + 42 - (12 + 14 - 12) = 45 - 30 = 15$

~~Ако~~ ако е отрицателна то следва че имат отрицателни посоки

$V_{ABCD} = \frac{1}{6}\left|\overrightarrow{BA}\,\overrightarrow{BC}\,\overrightarrow{BD}\right| = \frac{15}{6} = \frac{5}{3}$

$S_{ABCD} = \frac{1}{2}\left|\overrightarrow{BC} \times \overrightarrow{BD}\right| \Rightarrow$

$\overrightarrow{BC} \times \overrightarrow{BD} = \begin{vmatrix} \begin{pmatrix}-1 & 2\\ -7 & -3\end{pmatrix} & -\begin{pmatrix}-2 & 2\\ 4 & -3\end{pmatrix} & \begin{pmatrix}-2 & -1\\ 4 & -7\end{pmatrix}\end{vmatrix}$

$3 - 14 = -11 \qquad -(-6+8) = -2 \qquad -14 + 4 = -10$

$\left|\overrightarrow{BC} \times \overrightarrow{BD}\right| = \sqrt{11^2 + 2^2 + 10^2} = \sqrt{225} = 15$

$S = \frac{1}{2}\,15 = \frac{15}{2}$

$h_a = \frac{3V}{S} = \frac{3\frac{15}{2}}{\frac{15}{2}} = 3$

$|a| = |b| = |c| = 1$

$(ab)_\varepsilon = (bc)_\varepsilon = (ca)_\varepsilon = 60^\circ$

$[(a \times b) + c] \times [(b+c) + a][(c \times a) + b] =$

$[(a \times b) \times (b \times c) + (a \times b)a + c(b \times c) + ac][(c \times a) + b] =$

$[(abc)b - \underbrace{(abb)}_{0}c + a^2 b - (ab)a + c^2 b - (bc)c + c \times a][(c \times a) + b]$

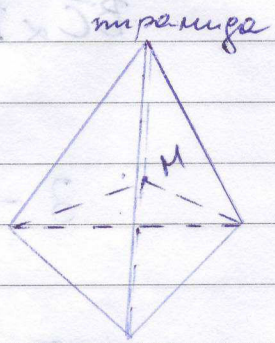
$(abc)^2 + (abc)b^2 + (abc)a^2 + a^2 b^2 - ab^2 + (abc)c^2 + c^2 b^2 - (bc)^2 +$
$(c \times a)^2 + abc = (abc)^2 + (a^2 + b^2 + c^2 + 1)(abc) + a^2 b^2 + b^2 c^2 + c^2 a^2 -$
$(ab)^2 + (bc)^2 + (ca)^2) \Rightarrow$

$\phi \quad (c \times a)^2 = a^2 c^2 - (ac)^2$

$\phi \quad (abc)^2 = \begin{vmatrix} a^2 & ab & ac \\ ba & b^2 & bc \\ ca & cb & c^2 \end{vmatrix} = \frac{1}{2} \Rightarrow abc = \pm \frac{\sqrt{2}}{2}$

по Грам

$\Rightarrow$ [illegible] $11/4 \pm 2\sqrt{2}$



$e_1 + e_2 + e_3 + e_4 = 0 \; ; \; e_1 + e_2 = -(e_3 + e_4) \uparrow^2$

$e_1^2 + e_2^2 + 2e_1e_2 = e_3^2 + e_4^2 + 2e_3e_4$

ка $M = 0 \Rightarrow e_1e_2 = e_3e_4$ / умножаваме по (-4) и местим горе

$e_1^2 + e_2^2 - 2e_1e_2 = e_3^2 + e_4^2 - 2e_3e_4$ / но това е равно на

$(e_1 - e_2)^2 = (e_3 - e_4)^2$ т.е. $|\overrightarrow{A_1A_2}|^2 = |\overrightarrow{A_3A_4}|^2$